Example

Seminar 7, pg. 2.

P_m _ Population proportion

$X_1 \ldots X_n$ $\quad n = 94$

$K = 50$

$\hat{P}_m = \frac{50}{94} \approx 0.53$

$Y_1 \ldots Y_m$ $\quad m = 68$

$r = 40$

$\boxed{\hat{P}_F = \frac{40}{68}} \approx 0.59$

P_F - population proportion

$$L < P_m - P_F < U$$

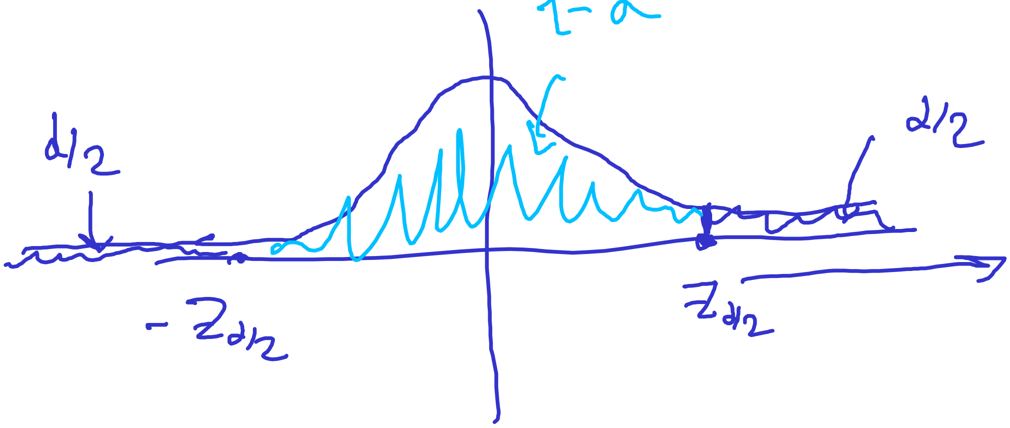
$$L = \hat{P}_m - \hat{P}_F - z_{\alpha/2} \cdot \sqrt{\frac{\hat{P}_m(1-\hat{P}_m)}{n} + \frac{\hat{P}_F(1-\hat{P}_F)}{m}}$$

$$U = \hat{P}_m - \hat{P}_F + z_{\alpha/2} \cdot \text{—— '' ——}$$

$$99\%\ CI = (1-\alpha)\cdot 100\%$$

$$\alpha = 0.01$$

$$\alpha/2 = 0.005$$



$$P(Z < z_{\alpha/2}) = 1 - \alpha/2 = 0.995$$

$$z_{\alpha/2} = 2.576$$

$$p_m - p_f \in \left(-0.06 - 2.576 \cdot \sqrt{\frac{0.53\cdot(1-0.53)}{94} + \frac{0.59\cdot(1-0.59)}{68}}\right.,$$

$$\left. -0.06 + 2.576 \cdot \text{—— '' ——} \right)$$

$X_1 \dots X_n$ $\qquad \frac{K}{n} = \hat{p}_1$

$n > 30$

$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$

$Y_1 \dots Y_m$, $\frac{r}{m} = \hat{p}_2$; $\hat{p}_2 \sim N\left(p_2, \frac{p_2(1-p_2)}{m}\right)$

$$\underbrace{\hat{p}_1 - \hat{p}_2}_{\hat{\theta}} \sim N\left(p_1 - p_2 ; \boxed{\frac{p_1(1-p_1)}{n} + \frac{p_2(1-p_2)}{m}}\right) \quad Var(\hat{\theta})$$

$$1-\alpha = P\left(L < \underbrace{p_1 - p_2}_{\theta} < U\right)$$

$$Var(X-Y) = Var(X) + Var(Y)$$

$$\underbrace{\frac{\hat{\Theta} - U}{\sqrt{var(\hat{\Theta})}}}_{-z_{\alpha/2}} < \underbrace{\frac{\hat{\Theta} - \Theta}{\sqrt{var(\hat{\Theta})}}}_{Z} < \underbrace{\frac{\hat{\Theta} - L}{\sqrt{var(\hat{\Theta})}}}_{z_{\alpha/2}}$$

$$z_{\alpha/2} = \frac{\hat{\Theta} - L}{\sqrt{var}} \quad ; \quad L = \hat{\Theta} - z_{\alpha/2} \cdot \sqrt{var \, \hat{\Theta}}$$

$$L = \hat{\Theta} - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{\hat{p}_2(1-\hat{p}_2)}{m}}$$

$$U = \hat{\Theta} + \quad -//-$$

CI: μ - ?

σ^2 is unknown.

χ^2 -distribution

$$\chi^2(k) = \sum_{i=1}^{k} (Z_i)^2 \quad ; \; k \text{ - degree of Freedom}$$

$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

Student's t-distribution

$$T_{(k)} = \frac{Z}{\sqrt{\frac{\chi^2(k)}{k}}} = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}\sqrt{\frac{S^2(n-1)}{\sigma^2(n-1)}}} = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim t_{(n-1 \text{ d.f.})}$$

$$S^2 = \frac{1}{n-1}\sum (X_i - \bar{X})^2 \quad ; \quad \frac{S^2 \cdot (n-1)}{\sigma^2} \sim \chi^2(n-1 \text{ d.f.})$$

$$1-\alpha = P(L < \mu < U)$$

$$\underbrace{\frac{\bar{X} - U}{\frac{S}{\sqrt{n}}}}_{-t_{\alpha/2}} < \underbrace{\frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}}_{t\,(n-1\ \text{d.f.})} < \underbrace{\frac{\bar{X} - L}{\frac{S}{\sqrt{n}}}}_{t_{\alpha/2}}$$

$$\boxed{\begin{aligned} L &= \bar{X} - t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} \\ U &= \bar{X} + t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} \end{aligned}}$$

Ex.

95% $\rightarrow$ $\alpha = 0.05$; $\alpha/2 = 0.025$

147, 84, 24, 85, 159

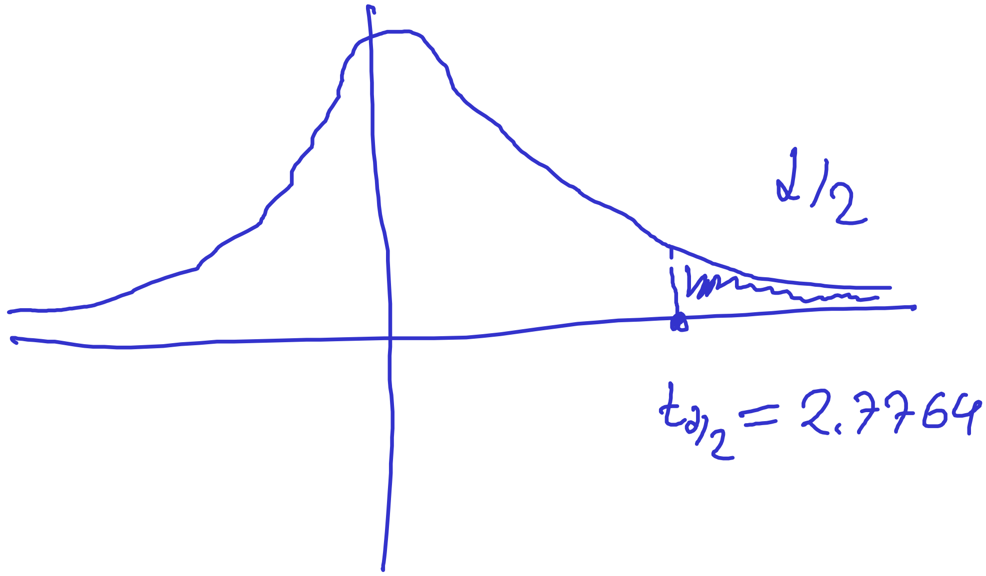
μ

σ^2 - unknown

$$\bar{X} = \frac{\sum X_i}{n} = 99.8$$

$$S^2 = \frac{1}{n-1} \cdot \sum (X_i - \bar{X})^2 = 2986.7$$

$$S = 54.65$$



$$\mu \in \left(99.8 - 2.7764 \cdot \frac{54.65}{\sqrt{5}} \; ; \; 99.8 + 2.7764 \cdot \text{-//-} \right)$$