② $\underbrace{\underline{X_1}, \underline{X_2}, \underline{X_3}}$ Sample. I.I.D Var...

$X \sim (\mu, \sigma^2)$

$E[X_1] = E[X_2] = E[X_3] = \mu$

$\hat{\sigma}^2 = c(X_1 - X_2)^2$ $\quad c - ?$

$\boxed{E[\hat{\sigma}^2] = \sigma^2}$

$\sigma^2 = E[X^2] - (E[X])^2$

$E[c(X_1 - X_2)^2] = \sigma^2$

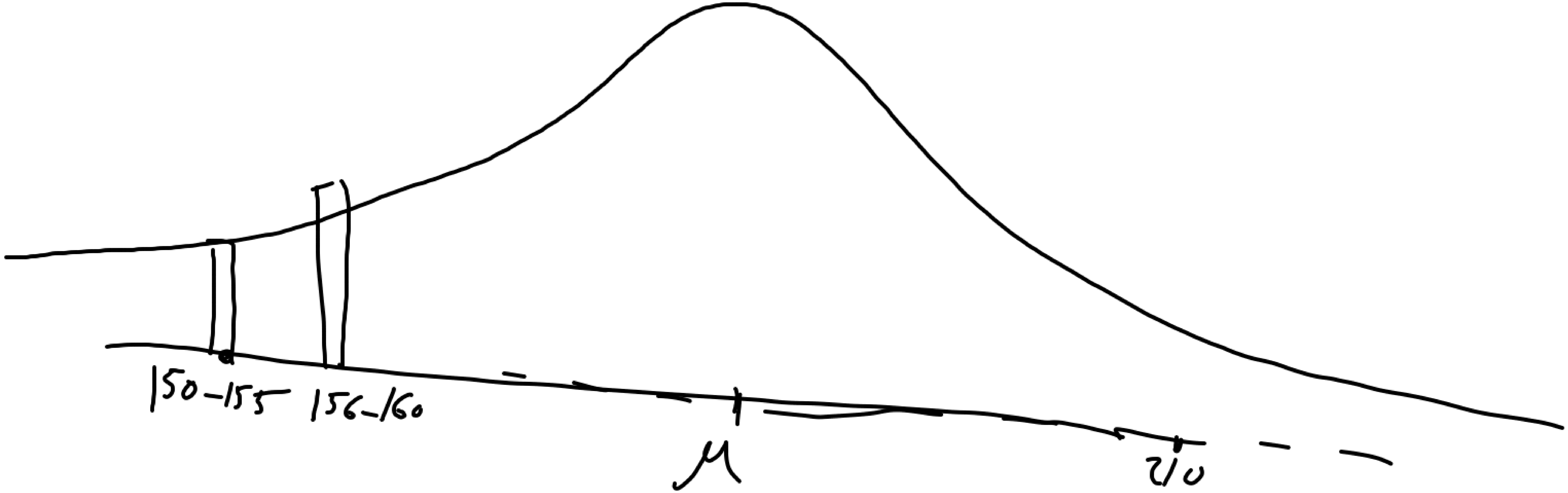
$\Rightarrow 2c\sigma^2 = \sigma^2$

$c = 1/2$.

$E[cX_1^2 - 2cX_1X_2 + cX_2^2] = cE[X_1^2] + cE[X_2^2] - 2cE[X_1X_2] =$

$= cE[X^2] + cE[X^2] - 2cE[X]E[X]$

$= 2c(\sigma^2 + \mu^2) - 2c\mu^2 = \sigma^2 \Rightarrow$



150-155
156-160
μ
210

$X_1 - - - - - X_n$ $X \sim (\mu, \sigma^2)$

I.I.D

$$\bar{X} = \frac{\sum X_i}{n}$$

$$Z_n = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$n = 10$

$n = 20$

$n = 30$

$F(z_n, n=30)$

$F(z_n, n=20)$

$F(z_n, n=10)$

$$Z_n = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow[n \to \infty]{} Z \sim N(0, 1)$$

$$\overline{X} \xrightarrow[n \to \infty]{} W \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$A \sim N(\mu, \sigma^2)$$

$$\frac{A - \mu}{\sigma} \sim N(0, 1)$$

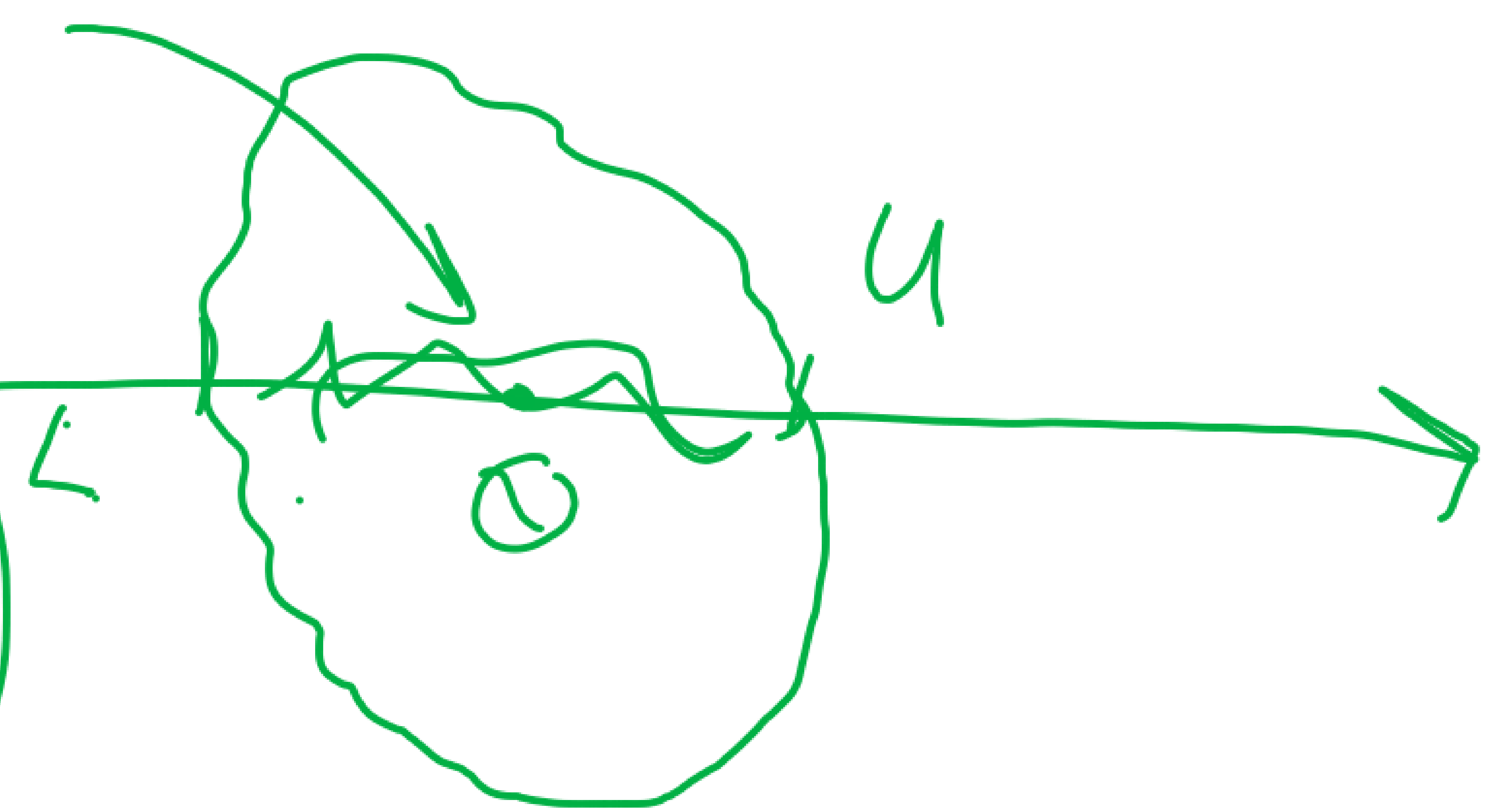
$$\hat{\Theta} = T(X_1 \dots X_n) \longrightarrow \Theta$$

$$P(\hat{\Theta} = \Theta) = 0$$

$1-\alpha$, $\alpha \in (0,1)$

Conf. level

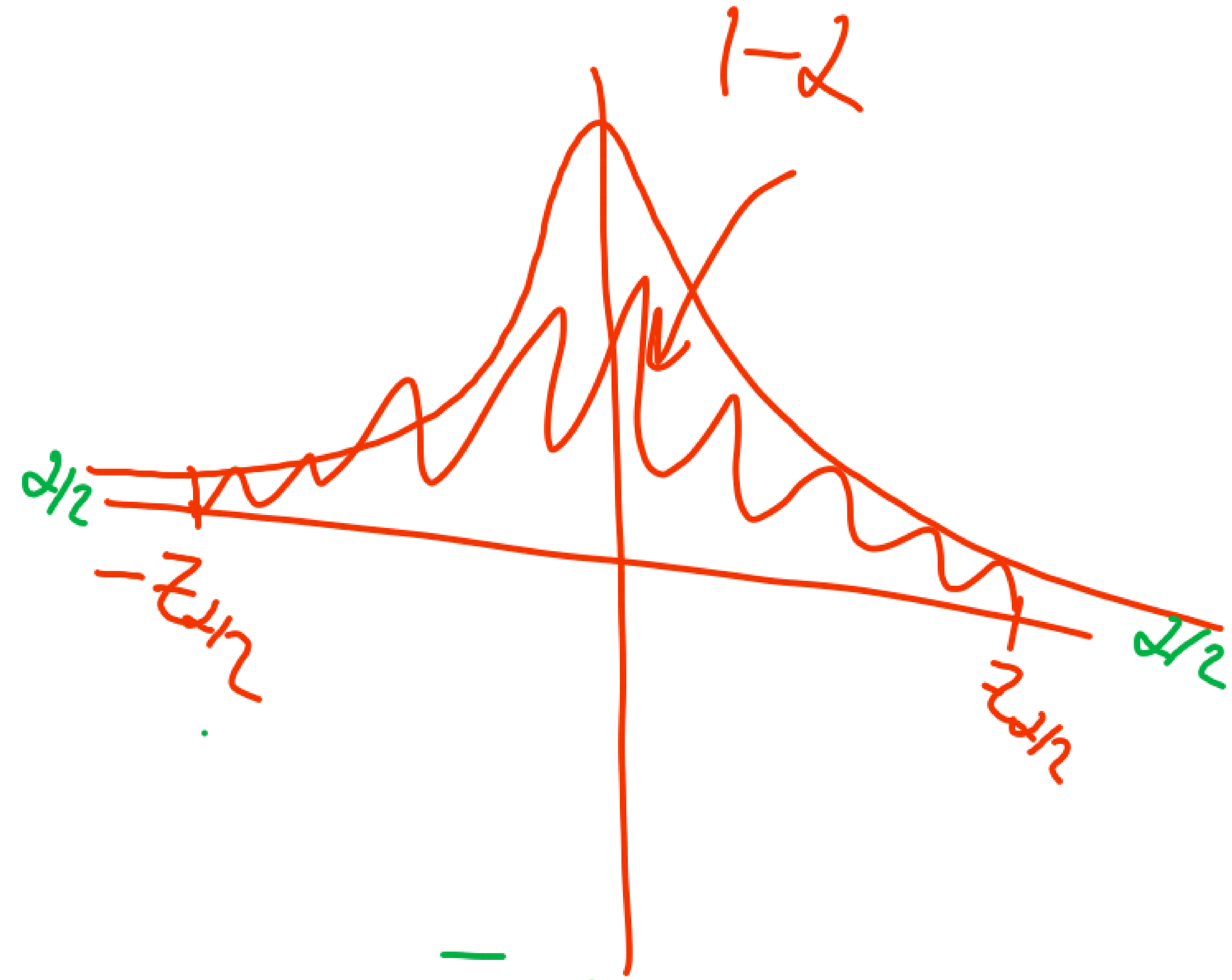
$$1-\alpha = P(L < \Theta < U)$$



μ, σ^2 - Known

$$1-\alpha = P(L < \mu < U)$$

$$\underbrace{\frac{\bar{X} - U}{\frac{\sigma}{\sqrt{n}}}}_{-z_{\alpha/2}} < \underbrace{\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}}_{Z} < \underbrace{\frac{\bar{X} - L}{\frac{\sigma}{\sqrt{n}}}}_{z_{\alpha/2}}$$



$$z_{\alpha/2} = \frac{\bar{X} - L}{\frac{\sigma}{\sqrt{n}}}$$

$$\underline{U = \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$$

$$\underline{L = \bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$$

$$L < \mu < U$$

Level of conf.

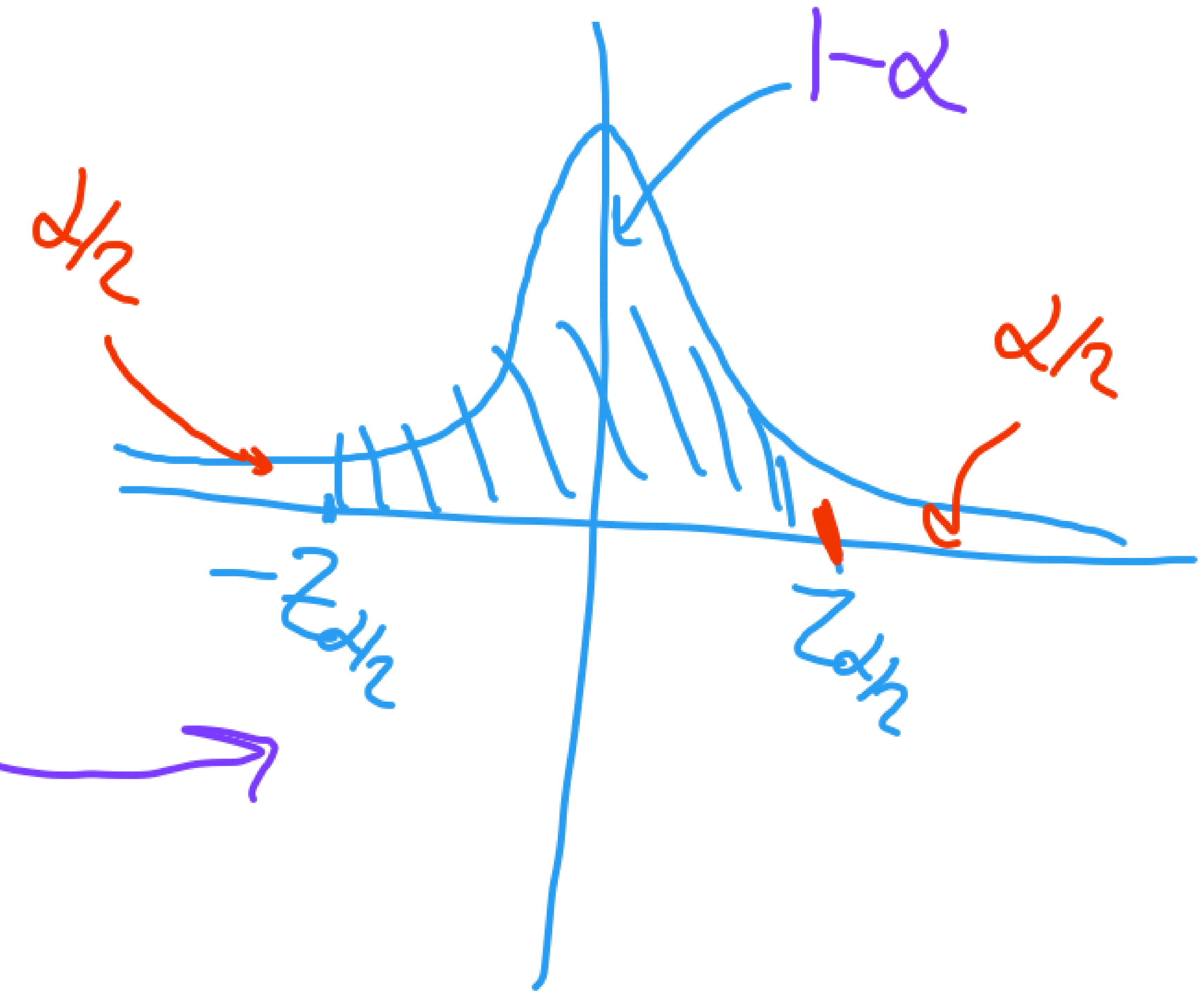
$$1-\alpha = P(L < \mu < U)$$

$$\alpha \in (0,1)$$

$$\underbrace{\frac{\bar{X}-U}{\sigma/\sqrt{n}}}_{= -Z_{\alpha/2}} < \underbrace{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}}_{Z} < \underbrace{\frac{\bar{X}-L}{\sigma/\sqrt{n}}}_{= Z_{\alpha/2}}$$

$$\frac{\bar{X}-L}{\sigma/\sqrt{n}} = Z_{\alpha/2} \Rightarrow \boxed{L = \bar{X} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$$

$$\boxed{U = \bar{X} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$$



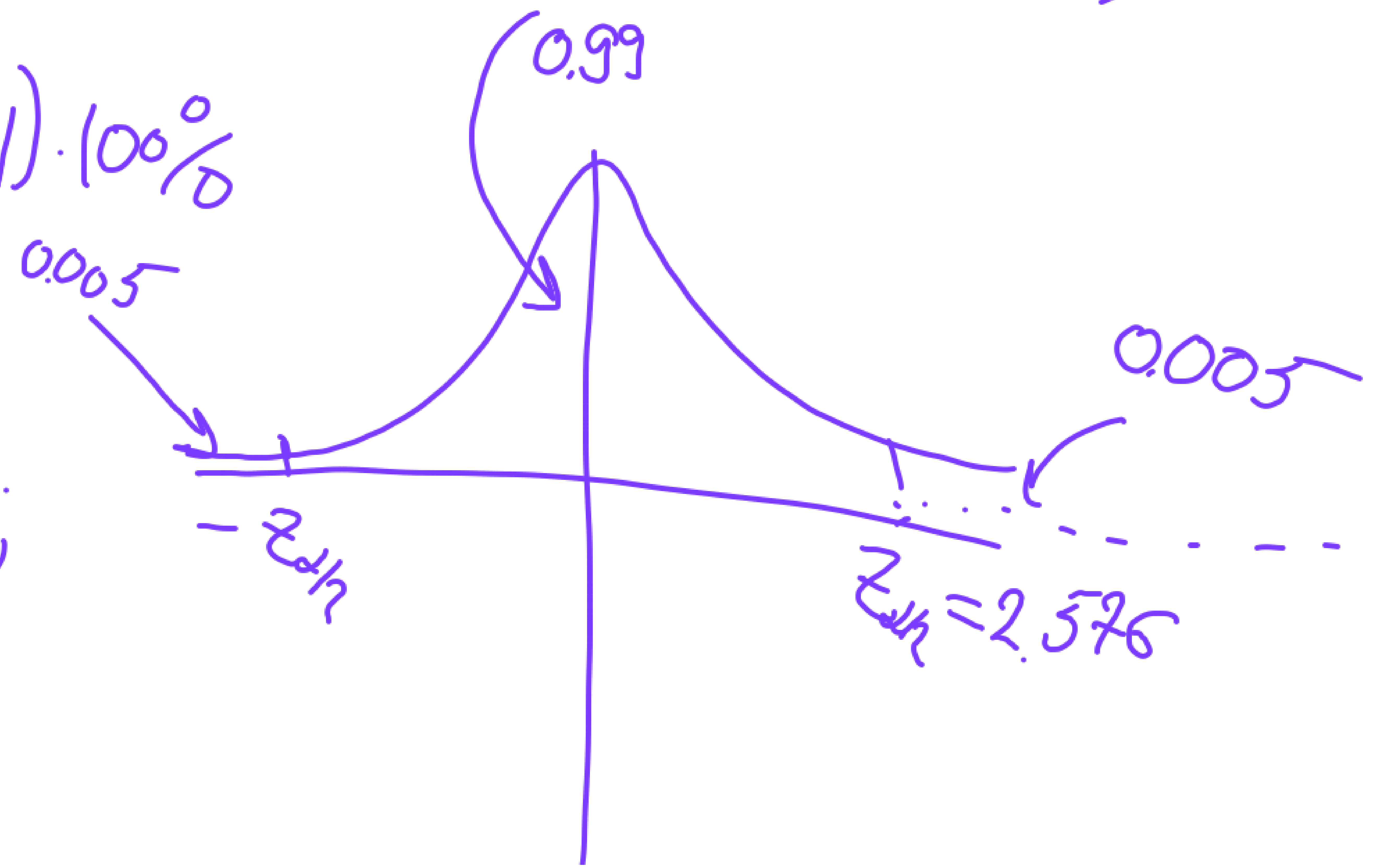
① 40 ; $\boxed{\sigma = 12.5}$

$\bar{X} = 76.3$

$$\mu \in \left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} ; \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$99\% = (1 - 0.01)\cdot 100\%$

$\alpha = 0.01$



$$\mu \in \left(76{,}3 - 2.576\cdot\frac{12.5}{\sqrt{40}} ; \; 76{,}3 + 2.576\,\frac{12.5}{\sqrt{40}}\right)$$