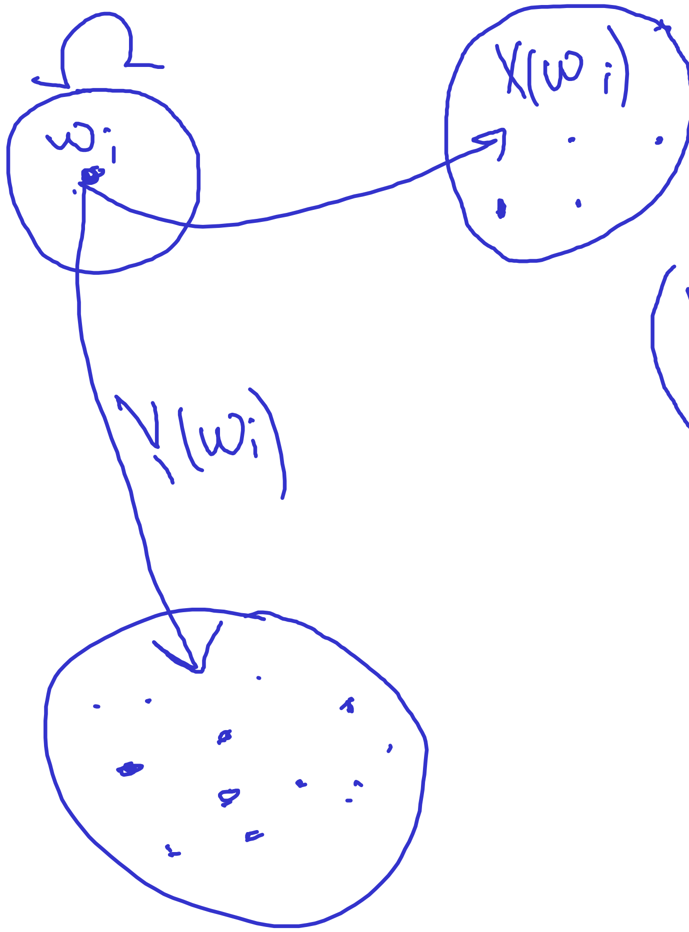
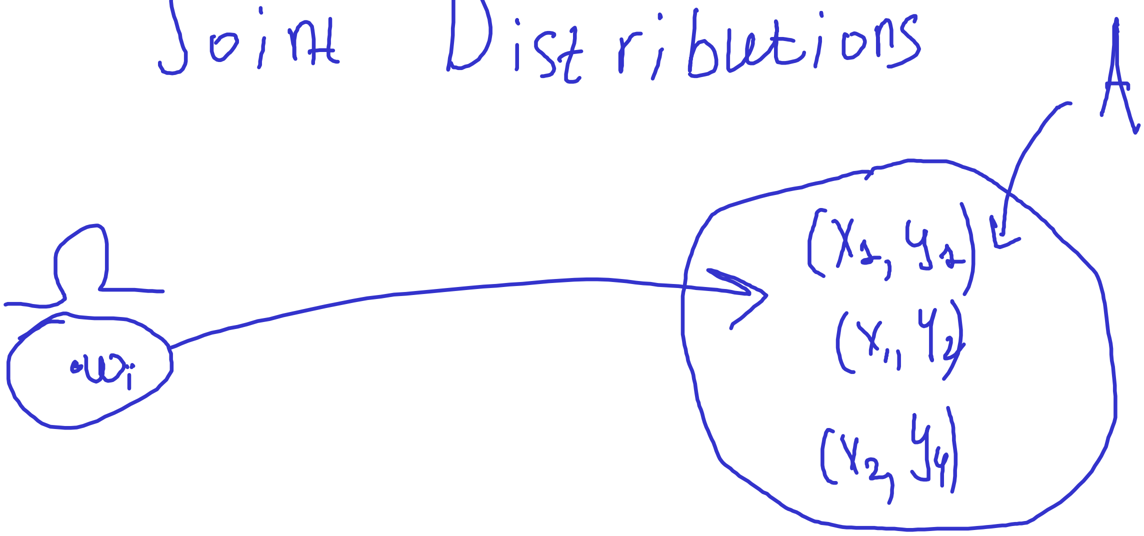


$$(X = x_1 ; Y = y_5)$$

Joint Distributions



Joint PMF

$$P(\{X = x_i ; Y = y_j\}) = p_{ij}$$

	$Y=y_1$	$Y=y_2$	$Y=y_3$
$X=x_1$	P_{11}	P_{12}	P_{13}
$X=x_2$			
$X=x_3$			

$$|A| = |X| \cdot |Y|$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} P(\{X=x_i;\, Y=y_j\}) = 1$$

	$Y=1$	$Y=-1$	
$X=2$	0.2	0.1	$P(X=2)=0.3$
$X=-1$	0.6	0.1	0.7
	$P(Y=1)=0.8$	0.2	

$P(Y=1 \mid X=2) = \frac{0.2}{0.3}$

$P(Y=-1 \mid X=2) = \frac{0.1}{0.3}$

$\frac{2}{3} + \frac{1}{3} = 1$

$$P(X=2) = P\left(\{X=2; Y=1\} \cup \{X=2; Y=-1\}\right) =$$

$$= \sum_{j=1}^{2} P\{X=2; Y=y_j\}$$

$$P(X=-1) = 0.6 + 0.1 = 0.7$$

Marginal PMF

Independence

$$P\{X=x_i ; Y=y_j\} = P(X=x_i) \cdot P(Y=y_j)$$

$$\boxed{E[g(x,y)] = \sum_{i=1}^{n} \sum_{j=1}^{m} g(x_i, y_j) \cdot P\{X=x_i ; Y=y_j\}}$$

$$E[X] = \sum_{i=1}^{n} x_i \cdot P\{X = x_i\}$$

Conditional PMF

$$P\{X = x_i \mid Y = y_j\} = \frac{P\{X = x_i ; Y = y_j\}}{P\{Y = y_j\}}$$

$$\sum_{i=1}^{n} P\{X = x_i \mid Y = y_j\} = ? = 1$$

№ 6

	$Y=0$	$Y=1$		
$X=0$	0.4	0.2		$P(X=0)=0.6$
$X=1$	0.1	0.3		0.4
	0.5	0.5		

$X=0,\ Y=0$

$0{,}4 \neq 0{,}5 \cdot 0{,}6$

$$P(X=0 \mid Y=1) = \frac{0.2}{0.5} = 2/5$$

$$P(X=1 \mid Y=1) = \frac{0.3}{0.5} = 3/5$$

№ 3

marginals

	$Y=-2$	$Y=-1$	$Y=3$	
$X=-2$	0.15	0.15	$c=0.3$	$P(X=-2)=0.6$
$X=1$	0.05	0.2	0.15	0.4
	0.2	0.35	0.45	

marginals

$$c = 1 - 0.3 - 0.2 - 0.2 = 0.3$$

$$P\{X=-2; Y=-2\} = P\{X=-2\} \cdot P\{Y=-2\}$$

$$0.15 \neq 0.6 \cdot 0.2$$

$$Cov(X,Y) = E[X \cdot Y] - E[X] \cdot E[Y]$$

$$E[X \cdot Y] = \sum_{i=} \sum_{j=} x_i y_i \cdot P\{X = x_i; Y = y_j\}$$

$$-2(-2) \cdot 0{,}15 + (-2) \cdot (-1) \cdot 0.15 + (-2) \cdot 3 \cdot 0{,}35$$

$$+ 1 \cdot (-2) \cdot 0{,}05 + 1 \cdot (-1) \cdot 0{,}2 + 1 \cdot 3 \cdot 0.15$$

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{var}(X)\cdot \mathrm{var}(Y)}}$$

$$-1 \leq \mathrm{corr}(X,Y) \leq 1$$

$\mathrm{corr} = 1 \qquad Y = \alpha X + \beta \quad , \quad \alpha > 0$

$\mathrm{corr} = -1 \qquad Y = \alpha X + \beta \quad , \quad \alpha < 0$

if X indep. from Y $\xLeftarrow{\times}\!\!\Rightarrow$ $\mathrm{corr}(X,Y) = \mathrm{cov}(X,Y) = 0$

$$P\{X=-2;\ Y<0\} = P(\{-2;-2\} \cup \{-2; Y=-1\}) =$$

$$= P\{X=-2; Y=-2\} + P\{X=-2; Y=-1\} = 0.3$$

$$P\{Y>-1\} = P(\{-2;3\} \cup \{1;3\}) = 0.45$$

$$P\{Y > X\} = P\left(\{-2;-1\} \cup \{-2;3\} \cup \{1;3\}\right) =$$

$$= 0.15 + 0.3 + 0.15 = \underline{0.5}.$$