Mathematical Statistics. Home Assignment 1.

Point estimation and confidence intervals.

MDI, Fall 2022.

1. The scores on a verbal reasoning test are normally distributed with a population mean of $\mu = 100$ and a population standard deviation of $\sigma = 10$.

   (a) What is the probability that a randomly chosen person scores at least 105?

   (b) A simple random sample of size $n = 20$ is selected. What is the probability that the sample mean will be between 97 and 104? (You may use the nearest values provided in the statistical tables.)

2. Let p be equal to the proportion of New York City residents who feel that the quality of life in New York City has become worse in the past few years. How large a sample is required to be 98% confident that the maximum error of the estimate of p is 0.025? Note: the error of the estimate is a half-width of the Confidence Interval, *i.e.* $z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

3. To determine the average age of its customers, a large manufacturer of clothing took a random sample of 50 customers and found $\bar{x} = 36$. If you know, that population standard deviation $\sigma = 12$:

   (a) Find a 95% confidence interval for the mean age μ of all customers,

   (b) Suppose you need to make the 95% confidence interval narrower, say ± 2 years. How large a sample is now required?

4. A random sample of 200 students is observed. 30 of them say they are 'really enjoying' Statistics.

   (a) Calculate the proportion of students in this sample saying they are 'really enjoying' Statistics and then provide a 95% confidence interval for the population proportion.

   (b) You now take a further random sample, in another institution. This time there are 40 students and 16 say they are 'really enjoying' Statistics. Provide a 95% confidence interval for this value. Think about why the two confidence intervals are different.

5. A car rental company is interested in the amount of time its vehicles are out of operation for repair work. A random sample of nine cars showed that over the past year, the numbers of day, each had been inoperative were

   16 10 21 22 8 17 19 14 19

   (a) Find the sample mean and the variance,

(b) Stating any assumptions you need to make, find a 90% confidence interval for the mean number of days in a year that vehicles in the company's fleet are out of the operation.

6. There is a concern about the speed of automobiles travelling over a particular part of the highway. For a random sanple of 7 automobiles, radar indicated the following speeds, in miles per hour:

$$79 \quad 73 \quad 68 \quad 77 \quad 86 \quad 71 \quad 69$$

   (a) Find the sample mean and the variance,

   (b) Assuming a normal population distribution, find a 95% confidence interval for the mean speed of all automobiles travelling over this part of the highway.

7. Suppose, that for a random sample of size n, a statistic $W(X_1, \ldots, X_n)$ is unbiased estimator for the population variance σ^2, and that W itself has a variance $\mathrm{Var}(W) = \frac{2\sigma^4}{(n-1)}$. Find for what value of the constant c the estimator of σ^2 of the form $T = cW$ has minimum mean squared error.

Bonus

Two measurements of the side of the square were produced. Suppose the two measurements X_1 and X_2 are independent random variables with mean a and variance σ^2. The true length of the side of the square is a. Find mean squared error for the following estimator of the area of the square: $T = X_1 X_2$.