$$Z_1^2 + Z_2^2 + Z_3^2 + \dots + Z_k^2 \sim \chi^2(k)$$

Chi-squared distr.

with k degrees of freedom

each Z_i - standard normal.

$$Y \sim N(\mu, \sigma^2)$$

$$\left(\frac{Y_1 - \mu}{\sigma}\right)^2 + \dots + \left(\frac{Y_k - \mu}{\sigma}\right)^2 \sim \chi^2_{(k)}$$

Example Sample $Y_1 \dots X_n$: μ, σ^2

$$Z = \frac{(\bar{X} - \mu)}{\sigma/\sqrt{n}}$$ - normal by CLT

Z^2 has chi-squared distr. w. 1 d.f.

d.f. − # of indep. observations

available to compute statistic

$S^2 = \sum \frac{x_i - \bar{x}}{(n-1)}$ d.f. $= n-1$

Ex. $X = 1\ 2\ 3$ $\bar{X} = 2$

$\sum x_i - \bar{x} = 0$ always (*)

deviations $x_1 - \bar{x}$, $x_2 - \bar{x}$, $x_3 - \bar{x}$

Only 2 can be chosen freely

(*) need to be satisfied.

$$\sum_{i=1}^{n} z_i^2 = \sum_{i=1}^{n} (z_i - \bar{z})^2 + n\bar{z}^2$$

$\sqrt{n}\cdot\bar{z}$ – normal S.V.

$n\bar{z}^2 \sim Chi(1)$

if $S^2 = \frac{\sum(z_i - \bar{z})^2}{n-1} \rightarrow S^2\cdot(n-1)$ has $\sim Chi(n-1)$

Let $X_1 \ldots X_n$ sample μ, σ^2 (not standard)

$$\boxed{\frac{S^2(n-1)}{\sigma^2}} = \frac{1}{\sigma^2}\sum(X_i - \bar{X})^2$$ has Chi^2 w. $n-1$ d.F.

$$= \frac{1}{\sigma^2}\sum\left(\frac{X_i - \mu}{\sigma} - \frac{\bar{X} - \mu}{\sigma}\right)^2 = \frac{1}{\sigma^2}\sum(z_i - \bar{z})^2$$

Def: Z - std normal r.v

χ^2

$$t = \frac{Z}{\sqrt{\frac{\chi^2}{d.f.}}} = \frac{Z}{\sqrt{\frac{\sum Z_i^2}{\nu}}}$$

t. distr. with ν d.f. !

$$t = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}} \sqrt{\frac{(n-1)\,S^2}{\sigma^2 (n-1)}}} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \quad !$$

Ex.

$$T = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{s^2(n-1)}{\sigma^2 \cdot (n-1)}}} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \sim t\,(\overset{10}{\cancel{n}} - 1)$$

$$\chi^2(n-1) = \frac{S^2(n-1)}{\sigma^2}$$