Point estimator brief recall

Recall:

- We say that $\bar{X}$ s a point estimate of the population mean
- We want our estimator to be unbiased (or low bias). Estimator $\bar{X}$ is unbiased provided our sample is random.
- We want our estimator to have a small variance (standard error)
- An efficient estimator is one that has lower MSE than another estimator

Errors in our estimate of the mean

- How large might $\bar{X} - \mu$ get?
- By the CLT, for the moderately large (>30) sample $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$
- Let's look at the two boundary points, $\pm z_{\alpha/2}$:

$$\pm z_{\alpha/2} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \rightarrow \mu = \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

- The probability that the true mean will fall within this range is $1{-}\alpha$

Confidence intervals

- A plausible range of values for the population parameter is called a confidence interval.
- Using only a point estimate to estimate a parameter is like fishing in a murky lake with a spear, and using a confidence interval is like fishing with a net.
- We can throw a spear where we saw a fish but we are more likely to miss. If we toss a net in that area, we have a better chance of catching the fish.
- If we report a point estimate, we probably will not hit the exact population parameter. If we report a range of plausible values – a confidence interval – we have a good shot at capturing the parameter.

Confidence intervals

Estimating population mean, μ

Population variance -- known

What do you need?

- Random Sample
- Population variance, σ^2, -- is known (!)
- $n > 30$ - CLT works fine, if not - assumption that population is normally distributed,

Point estimate:

$\bar{X} (sample mean) is an unbiased point estimator for population mean

Margins of errors

One of possible ways to write:

$E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Can be thought of as Max difference between $\bar{x}$ and μ.

$\bar{X} - E < \mu < \bar{X} + E$

Construction of CI

- Check requirements
- Calculate critical value $Z_{\alpha/2}$
- Find out the Error margin E, $E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
- Make confidence interval

Example 1

Random sample of 40 students. The average resting heart-rate for the sample was 76.3 bpm. Assume the population std is 12.5 bpm. Construct a 99% CI for the average resting heart-rate of the population.

Example 2

Problem C.2.1. Manager of a restaurant wants to estimate the mean amount *m* that a visitor spends for a lunch. A sample contains 36 visitors. Sample mean is $\bar{x} = \$3.60$. Manager knows that the standard deviation for one visitor is \$0.72. Find the confidence level corresponding to the interval (\$3.5; \$3.7).

Example 3

A college admission officer for an MBA program has determined that historically candidates have undergraduate grade point averages that are normally distributed with std 0.45. A random sample of 25 applications from the current year is taken, yielding a sample mean grade average of 2.90.

- Find a 95% CI for the populatin mean
- Based on these sample results, a statistician computes for the populatin mean a CI running from 2.81 to 2.99. Find the probability content associated with this interval.