Problem 4

HT

Route A $X \sim N(\mu_1, \sigma^2)$

Route B $Y \sim N(\mu_2, \sigma^2)$

$X_1 \ldots X_5$ $H_0: \mu_1 = \mu_2$

$Y_1 \ldots Y_5$ $H_1: \mu_1 \neq \mu_2$

don't know σ

$\hat{\theta} = \bar{X} - \bar{Y}$ $E(\hat{\theta}) = E(\bar{X} - \bar{Y}) =$

$X = X_1 \ldots X_n$ $\sigma_1^2 = \sigma_2^2 = \sigma^2$ $= E(\bar{X}) - E(\bar{Y}) = \mu_1 - \mu_2 = \theta$

$Y = Y_1 \ldots Y_m$

unbiase: $E[\hat{\theta}] = \theta$

$\mathrm{Var}(\hat{\theta}) = \mathrm{Var}(\bar{X} - \bar{Y}) = \mathrm{var}(\bar{X}) + \mathrm{var}(\bar{Y}) =$

$= \frac{\sigma^2}{n} + \frac{\sigma^2}{m} = \sigma^2 \cdot \left(\frac{1}{n} + \frac{1}{m}\right)$

$\mathrm{var}(\bar{X}) = \frac{\sigma^2}{n}$

CLT

$$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{X}-\mu}{\sqrt{Var(\bar{X})}} \sim N(0,1)$$

$$Z = \frac{\hat{\theta}-\theta}{\sqrt{var(\hat{\theta})}} = \frac{\hat{\theta}-\theta}{\underbrace{\sigma}_{\text{unknown}} \cdot \sqrt{\frac{1}{n}+\frac{1}{m}}}$$

$$t = \frac{Z}{\sqrt{\frac{\chi^2(k)}{k}}}$$

$$\frac{S^2(n-1)}{\sigma^2} \sim \chi^2(n-1\ d.f.)$$

$$\underbrace{\frac{S_1^2\cdot(n-1)}{\sigma^2}}_{\chi^2(n-1)} + \underbrace{\frac{S_2^2\cdot(m-1)}{\sigma^2}}_{\chi^2(m-1)} \sim \chi^2(n+m-2\ d.f.)$$

$$Z$$

$$t = \frac{\dfrac{\hat{\theta} - \theta}{\sigma \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}}}{\sqrt{\dfrac{S_1^2(n-1) + S_2^2(m-1)}{\sigma^2 (n+m-2)}}} =$$

pooles variance

$$S_p^2$$

$$= \frac{\hat{\theta} - \theta}{S_p \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}} \sim t_{(n+m-2\ d.f.)}$$

Problem 4.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$\overline{X} = 25 \quad ; \quad \overline{Y} = 29$

$$S^2 = \frac{1}{n-1} \sum (X_i - \overline{X})^2$$

$S_X = 5.92$

$S_Y = 5.61$

$\Theta = \mu_1 - \mu_2 = 0$

$$T = \frac{\hat{\Theta}}{S_p \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}}$$

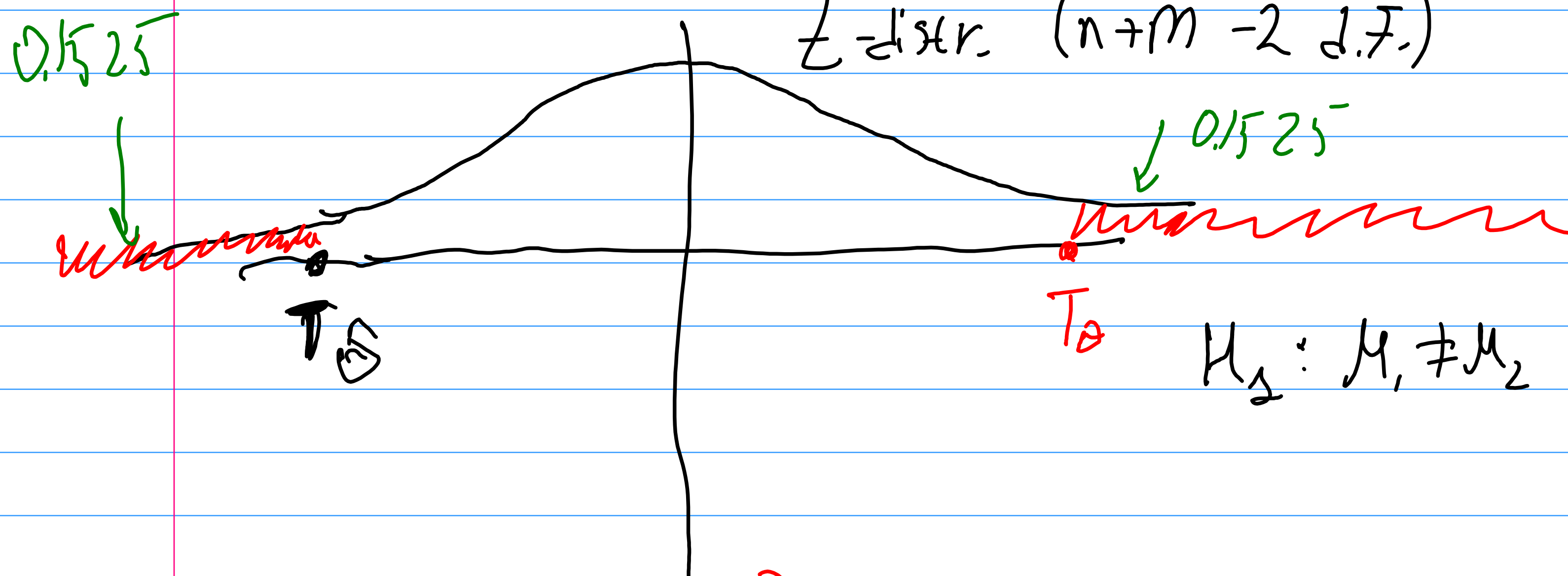
$$S_p = \sqrt{\frac{S_X^2 \cdot (n-1) + S_Y^2 (m-1)}{n+m-2}} =$$

$$= \sqrt{\frac{(5.92)^2 \cdot 4 + (5.61)^2 (4)}{5+5-2}} =$$

$= 5.77$

$$n = m = 5$$

$$T = \frac{\bar{X} - \bar{Y}}{S_p \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{-4}{5.77 \cdot \sqrt{\frac{2}{5}}} = -1.096$$



$H_1: \mu_1 \neq \mu_2$

p-val: $P(t \in (-\infty; -|T_0|)) + P(t \in (|T_0|; +\infty))$

p-value: 0.305 compare with

$\alpha = 0.05$

p-value $> \alpha \Rightarrow$ can not reject H_0

Problem 5

$H_0: \mu_2 = \mu_1 + 1$

$H_1: \mu_2 - \mu_1 \neq 1$

$X \leftarrow$ public $\quad n = 9 \quad \bar{X} = 55.78$

$Y \quad m = 7 \quad \bar{Y} = 56.28$

$\nearrow$ Private

$S_x^2 = 128.67$

$S_Y^2 = 530.9$

$$S_p = \sqrt{\frac{S_x^2(n-1) + S_Y^2(m-1)}{n+m-2}} =$$

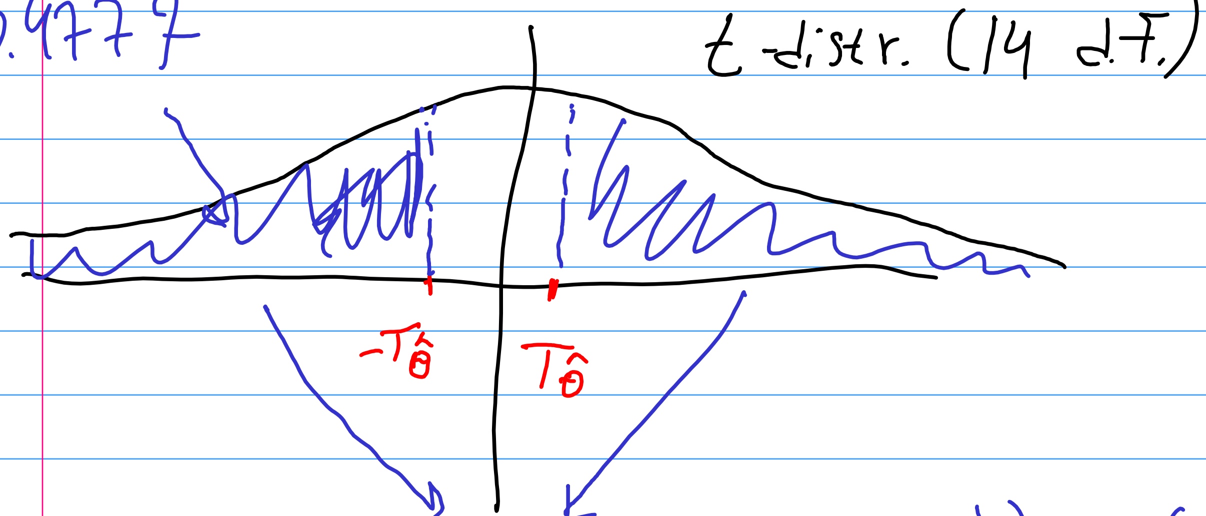
$$= \sqrt{\frac{(9-1)\cdot 128.67 + (7-1)\cdot 530.9}{9+7-2}} = \sqrt{301.05}$$

$$\Theta = \mu_1 - \mu_2 = 1$$

$$T_{\hat\theta} = \frac{\bar X - \bar Y - 1}{S_p \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}} =$$

$$= \frac{(57.78 - 56.28) - 1}{\sqrt{301.05} \cdot \sqrt{\frac{1}{9} + \frac{1}{7}}} = 0.057.$$

0.4777



$$\text{P-val} = P(t(-\infty, -T_{\hat\theta})) + P(t \in (T_{\hat\theta}, +\infty))$$

$$\text{P-val} \simeq 2 \cdot 0.4777 \gg 0.05$$

Can not reject H_0.