Class 4. Conditional probability. Total probability and Bayes' theorem.

Gleb Karpov

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1 Conditional probability

1. We toss a fair coin three successive times. Find the conditional probability $P(A|B)$ when A and B are the events: $A = \{\text{more heads than tails come up}\}$, $B = \{\text{1st toss is a head}\}$.

2. Radar Detection. If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present. the radar generates a (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.05. What is the probability of no aircraft presence and a false alarm? What is the probability of aircraft presence and no detection?

3. A fair six-sided die is thrown twice. Let B be the event that the first number thrown is no larger than 3, and let C be the event that the sum of the two numbers thrown equals 6. Find the probabilities of B and C, and the conditional probabilities of C given B, and of B given C.

2 Total probability and Bayes rule

1. If $P(\overline{B}) = 1/4$ and $P(A|B) = 1/2$, what is $P(A \cap B)$?

2. You enter a chess tournament where your probability of winning a game is 0.3 against half the players (call them type 1), 0.4 against a quarter of the players (call them type 2), and 0.5 against the remaining quarter of the players (call them type 3). You play a game against a randomly chosen opponent. What is the probability of winning?

3. Return to the chess problem. Suppose that you win. What is the probability $P(A_1|B)$ that you had an opponent of type 1?

4. You roll a fair four-sided die. If the result is 1 or 2, you roll once more but otherwise, you stop. What is the probability that the sum total of your rolls is at least 4?

5. (UoL Exam) The probability of a horse winning a race is 0.3 if it is dry and 0.5 if it is wet. The weather forecast gives the chance of rain as 40%.

   (a) Find the probability that the horse wins.

   (b) If you are told that the horse lost the race, what is the probability that the weather was dry on the day of the race? (Assuming you cannot remember!)

6. (UoL Exam) An engine encounters a standard environment with a probability of 0.95, and a severe environment with a probability of 0.05. In a normal environment the probability of failure is 0.02, whereas in the severe environment this probability is 0.5.

   (a) What is the probability of failure?

   (b) Given that failure has occurred, what is the probability that the environment encountered was severe?

7. A test for a certain rare disease is assumed to be correct 95% of the time. A random person drawn from a certain population has probability 0.01 of having the disease. Given that the person just tested positive, what is the probability of having the disease?

8. A doctor assumes that a patient has one of three diseases d_1, d_2, or d_3. Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has d_1 , probability 0.6 if he has disease d_2, and 0.4 if he has disease d_3 . Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?