Class 2. Probability and its classical definition. Counting Principles.

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1 Probability

1. A student must choose one of the subjects, art, geology, or psychology, as an elective. She is equally likely to choose art or psychology and twice as likely to choose geology. What are the respective probabilities that she chooses art, geology, and psychology?

2. For a bill to come before the president of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to these two bodies, 60 percent pass the House, 80 percent pass the Senate, and 90 percent pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president.

3. Let A and B be events such that $P(A \cap B) = 1/4$, $P(\overline{A}) = 1/3$, and $P(B) = 1/2$. What is $P(A \cup B)$?

4. Consider the experiment of rolling a pair of 4-sided dice. We assume the dice are fair, and we interpret this assumption to mean that each of the sixteen possible outcomes (pairs (i, j), with $i, j = 1, 2, 3, 4$) has the same probability of 1/16. Calculate probabilities of the following events:

   - Sum of the rolls is even
   - The first roll is equal to the second
   - The first roll is larger than the second
   - At least one roll is equal to 4.

5. A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as a two.) What is the probability of getting an even number in one throw?

6. Two cards are drawn successively from a collection of 10 cards: $\{1, \ldots, 5\}$ diamonds, and $\{1, \ldots, 5\}$ hearts. Find the probability that the second card is higher in rank than the first card.

2 Counting Principles

1. A college planning committee consists of 3 freshmen, 4 sophomores, 5 juniors, and 2 seniors. A subcommittee of 4, consisting of 1 person from each class, is to be chosen. How many different subcommittees are possible?

2. A local telephone number is a 7-digit sequence, but the first digit has to be different from 0 or 1. How many distinct telephone numbers are there?

3. A class consists of 6 men and 4 women. An examination is given, and the students are ranked according to their performance. Assume that no two students obtain the same score.

   - How many different rankings are possible?
   - If the men are ranked just among themselves and the women just among themselves, how many different rankings are possible?

4. Ms. Jones has 10 books that she is going to put on her bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Ms. Jones wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

5. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if:

(a) there are no other restrictions?

(b) 2 of the men refuse to serve together?

(c) 2 of the women refuse to serve together?

(d) 1 man and 1 woman refuse to serve together?